# Formularium goniometrie

**Definitie van $\sin\alpha$ en $\cos\alpha$ voor een willekeurige georiënteerde hoek $\alpha$:**

Voor een hoek $\hat{\alpha}$ heeft het beeldpunt $\alpha$ de coördinaten $(\cos\alpha, \sin\alpha)$.

**Definitieformules:**

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \cot\alpha = \frac{\cos\alpha}{\sin\alpha} \qquad \sec\alpha = \frac{1}{\cos\alpha} \qquad \csc\alpha = \frac{1}{\sin\alpha}$$

**Hoofdformule van de goniometrie met gevolgen:**

$$\sin^2\alpha + \cos^2\alpha = 1 \qquad 1 + \tan^2\alpha = \sec^2\alpha \qquad 1 + \cot^2\alpha = \csc^2\alpha$$

**Tabel van gekende exacte waarden:**

| $\alpha$ (in graden) | $0°$ | $30°$ | $45°$ | $60°$ | $90°$ |
|---|---|---|---|---|---|
| $\alpha$ (in radialen) | $0$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ |
| $\sin\alpha$ | $0$ | $\frac{1}{2}$ | $\frac{1}{2}\sqrt{2}$ | $\frac{1}{2}\sqrt{3}$ | $1$ |
| $\cos\alpha$ | $1$ | $\frac{1}{2}\sqrt{3}$ | $\frac{1}{2}\sqrt{2}$ | $\frac{1}{2}$ | $0$ |
| $\tan\alpha$ | $0$ | $\frac{1}{3}\sqrt{3}$ | $1$ | $\sqrt{3}$ | $\backslash$ |
| $\cot\alpha$ | $\backslash$ | $\sqrt{3}$ | $1$ | $\frac{1}{3}\sqrt{3}$ | $0$ |

**Verwante hoeken:**

| Gelijke beeldpunten | Tegengestelde hoeken | Supplementaire hoeken |
|---|---|---|
| $\sin(\alpha + k\cdot 2\pi) = \sin\alpha$ | $\sin(-\alpha) = -\sin\alpha$ | $\sin(\pi - \alpha) = \sin\alpha$ |
| $\cos(\alpha + k\cdot 2\pi) = \cos\alpha$ | $\cos(-\alpha) = \cos\alpha$ | $\cos(\pi - \alpha) = -\cos\alpha$ |
| $\tan(\alpha + k\cdot 2\pi) = \tan\alpha$ | $\tan(-\alpha) = -\tan\alpha$ | $\tan(\pi - \alpha) = -\tan\alpha$ |
| $\cot(\alpha + k\cdot 2\pi) = \cot\alpha$ | $\cot(-\alpha) = -\cot\alpha$ | $\cot(\pi - \alpha) = -\cot\alpha$ |
| $\sec(\alpha + k\cdot 2\pi) = \sec\alpha$ | $\sec(-\alpha) = \sec\alpha$ | $\sec(\pi - \alpha) = -\sec\alpha$ |
| $\csc(\alpha + k\cdot 2\pi) = \csc\alpha$ | $\csc(-\alpha) = -\csc\alpha$ | $\csc(\pi - \alpha) = \csc\alpha$ |

| Complementaire hoeken | Anti-complementaire hoeken | Anti-supplementaire hoeken |
|---|---|---|
| $\sin(\frac{\pi}{2} - \alpha) = \cos\alpha$ | $\sin(\frac{\pi}{2} + \alpha) = \cos\alpha$ | $\sin(\pi + \alpha) = -\sin\alpha$ |
| $\cos(\frac{\pi}{2} - \alpha) = \sin\alpha$ | $\cos(\frac{\pi}{2} + \alpha) = -\sin\alpha$ | $\cos(\pi + \alpha) = -\cos\alpha$ |
| $\tan(\frac{\pi}{2} - \alpha) = \cot\alpha$ | $\tan(\frac{\pi}{2} + \alpha) = -\cot\alpha$ | $\tan(\pi + \alpha) = \tan\alpha$ |
| $\cot(\frac{\pi}{2} - \alpha) = \tan\alpha$ | $\cot(\frac{\pi}{2} + \alpha) = -\tan\alpha$ | $\cot(\pi + \alpha) = \cot\alpha$ |
| $\sec(\frac{\pi}{2} - \alpha) = \csc\alpha$ | $\sec(\frac{\pi}{2} + \alpha) = -\csc\alpha$ | $\sec(\pi + \alpha) = -\sec\alpha$ |
| $\csc(\frac{\pi}{2} - \alpha) = \sec\alpha$ | $\csc(\frac{\pi}{2} + \alpha) = \sec\alpha$ | $\csc(\pi + \alpha) = -\csc\alpha$ |

**Som- en verschilformules:**

$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta \qquad \sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta \qquad \cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta} \qquad \tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\cdot\tan\beta}$$

**Verdubbelingsformules:**

$$\sin 2\alpha = 2\cdot\sin\alpha\cdot\cos\alpha \qquad \tan 2\alpha = \frac{2\cdot\tan\alpha}{1-\tan^2\alpha}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\cdot\sin^2\alpha = 2\cdot\cos^2\alpha - 1$$

**Andere formules:**

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha \qquad \cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$

**De formules van Carnot:**

$$\cos^2\alpha = \frac{1+\cos(2\alpha)}{2} \qquad \sin^2\alpha = \frac{1-\cos(2\alpha)}{2}$$

**De t-formules:**

$$\sin\alpha = \frac{2\cdot\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{2t}{1+t^2} \qquad \cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{1-t^2}{1+t^2} \qquad \tan\alpha = \frac{2\cdot\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{2t}{1-t^2}$$

**De formules van Simpson (som naar product):**

$$\sin\alpha+\sin\beta = 2\cdot\sin\tfrac{1}{2}(\alpha+\beta)\cdot\cos\tfrac{1}{2}(\alpha-\beta) \qquad \sin\alpha-\sin\beta = 2\cdot\cos\tfrac{1}{2}(\alpha+\beta)\cdot\sin\tfrac{1}{2}(\alpha-\beta)$$

$$\cos\alpha+\cos\beta = 2\cdot\cos\tfrac{1}{2}(\alpha+\beta)\cdot\cos\tfrac{1}{2}(\alpha-\beta) \qquad \cos\alpha-\cos\beta = -2\cdot\sin\tfrac{1}{2}(\alpha+\beta)\cdot\sin\tfrac{1}{2}(\alpha-\beta)$$

**De formules van Simpson (product naar som):**

$$\sin\alpha\cdot\sin\beta = \tfrac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta)) \qquad \cos\alpha\cdot\sin\beta = \tfrac{1}{2}(-\sin(\alpha-\beta)+\sin(\alpha+\beta))$$

$$\sin\alpha\cdot\cos\beta = \tfrac{1}{2}(\sin(\alpha-\beta)+\sin(\alpha+\beta)) \qquad \cos\alpha\cdot\cos\beta = \tfrac{1}{2}(\cos(\alpha-\beta)+\cos(\alpha+\beta))$$

**Elementaire goniometrische vergelijkingen:**

$$\sin x = \sin\alpha \iff x = \alpha + 2k\pi \;\vee\; x = \pi - \alpha + 2k\pi$$

$$\cos x = \cos\alpha \iff x = \alpha + 2k\pi \;\vee\; x = -\alpha + 2k\pi$$

$$\tan x = \tan\alpha \iff x = \alpha + k\pi$$

**Bijzondere gevallen:**

| | | |
|---|---|---|
| $\sin x = 0 \iff x = k\pi$ | $\cos x = 0 \iff x = \frac{\pi}{2} + k\pi$ | $\tan x = 0 \iff x = k\pi$ |
| $\sin x = 1 \iff x = \frac{\pi}{2} + 2k\pi$ | $\cos x = 1 \iff x = 2k\pi$ | $\tan x = 1 \iff x = \frac{\pi}{4} + k\pi$ |
| $\sin x = -1 \iff x = \frac{3\pi}{2} + 2k\pi$ | $\cos x = -1 \iff x = \pi + 2k\pi$ | $\tan x = -1 \iff x = \frac{3\pi}{4} + k\pi$ |